



MODELING INDOOR PARTICLE DEPOSITION FROM TURBULENT FLOW ONTO SMOOTH SURFACES

Alvin C. K. Lai and William W. Nazaroff*

Indoor Environment Department, Environmental Energy Technologies Division, Ernest Orlando Lawrence Berkeley National Laboratory, Berkeley, CA 94720, U.S.A. and Department of Civil and Environmental Engineering, University of California, Berkeley, CA 94720-1710, U.S.A.

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Abstract—Particle deposition to indoor surfaces is frequently modeled by assuming that indoor air flow is homogeneously and isotropically turbulent. Existing formulations of such models, based on the seminal work of Corner and Pendlebury (1951, *Proc. Phys. Soc. Lond.* **B 64**, 645), lack a thorough physical foundation. We apply the results of recent studies of near-surface turbulence to produce an analogous model for particle deposition onto indoor surfaces that remains practical to use yet has a stronger physical basis. The model accounts for the effects of Brownian and turbulent diffusion and gravitational settling. It predicts deposition to smooth surfaces as a function of particle size and density. The only required input parameters are enclosure geometry and friction velocity. Model equations are presented for enclosures with vertical and horizontal surfaces, and for spherical cavities. The model helps account for a previously unexplained experimental observation regarding the functional dependence of deposition velocity on particle size. Model predictions agree well with recently published experimental data for a spherical cavity (Cheng, Y. S., *Aerosol Sci. Technol.* **27**, 131–146, (1997)). © 2000 Elsevier Science Ltd. All rights reserved

1. INTRODUCTION

Inhalation exposure to airborne particles can have adverse effects on health. Since people spend most of their time indoors (Jenkins *et al.*, 1992), an important air quality issue is the presence of particles in indoor air (Wallace, 1996; Özkaynak *et al.*, 1996). Such particles may have penetrated the building shell from outdoor air or may have been emitted directly from indoor sources such as tobacco smoking, cooking, occupants, building materials, and consumer products.

One fate for particles in indoor air is deposition onto surfaces. Clearly, this process alters the likelihood of human exposure, since a deposited particle cannot be inhaled unless resuspended. On the other hand, particle deposition may lead to material degradation such as soiling of artworks (Nazaroff and Cass, 1991) or damage to electronic equipment (Weschler *et al.*, 1996). Knowledge of the rates of particle deposition onto indoor surfaces and the factors governing those rates is therefore important with respect to multiple indoor air quality concerns.

One widely used approach for modeling particle deposition to indoor surfaces assumes that the air flow is homogeneously and isotropically turbulent. In most realizations of this approach, the particle concentration is assumed to be uniform throughout the room except in thin boundary layers adjacent to room surfaces. Particles migrate through the boundary layers by turbulent and Brownian diffusion plus (for horizontal surfaces) gravitational settling.

Table 1 summarizes model developments for predicting particle deposition from turbulent enclosure flow. Corner and Pendlebury (1951) derived the first analytical expression for particle deposition in a rectangular enclosure under homogeneously turbulent flow. An important assumption in their version of the model is that, in the particle concentration boundary layer, the particle turbulent (eddy) diffusivity, ε_p , has the form

$$\varepsilon_p = K_e y^2, \quad (1)$$

* Author to whom correspondence should be addressed.

Table 1. Summary of model developments for particle deposition from turbulent flow in enclosures

Investigators	Expressions	Comments
Corner and Pendlebury (1951)	$\varepsilon_p = K_e y^2$ $K_e = \kappa^2 \frac{dU}{dy}$	The first published analytical work. Velocity gradient, dU/dy , evaluated based on a drag-force balance for a flat plate.
Crump and Seinfeld (1981)	$\varepsilon_p = K_e y^n$ $K_e = 0.4 \frac{dU}{dy}$	Exponent n is arbitrary. Analyzed overall depositional loss for vessel of arbitrary shape. Velocity gradient, dU/dy , evaluated based on energy dissipation rate (Okuyama <i>et al.</i> , 1977).
McMurry and Rader (1985)	$\varepsilon_p = K_e y^2$	Extension of Crump and Seinfeld (1981) theory to include electrostatic drift. Turbulence intensity parameter, K_e , was treated as an empirical parameter obtained by fitting experimental results.
Nazaroff and Cass (1989)	$\varepsilon_p = K_e y^2$ $K_e = \kappa^2 \frac{dU}{dy}$	Follows work of Corner and Pendlebury, incorporating the effects of thermophoresis.
Shimada <i>et al.</i> (1989)	$\varepsilon_p = K_e y^{2.7}$ $K_e = 7.5 \sqrt{\frac{2\varepsilon}{15\nu}}$	Incorporates the effect of particle inertia.
Beneš and Holub (1996)	$\varepsilon_p = K_e \delta^2 \left(\frac{y}{\delta}\right)^n$ $\delta = \frac{L}{\sqrt{\text{Re}}}$ $K_e = \kappa^2 \frac{dU}{dy}$	Eliminates dimensional problems associated with non-integer value of n . Expression for δ based on theory for laminar boundary layer; Re based on the velocity at tip of stirrer blade. Velocity gradient, dU/dy , evaluated based on work of Okuyama <i>et al.</i> (1986).

where K_e is a turbulence intensity parameter and y is the distance from the surface. This expression is based on Prandtl’s mixing length model for turbulent motion. Later, Crump and Seinfeld (1981) extended the Corner and Pendlebury model to predict the particle deposition loss-rate coefficient for an enclosure of arbitrary shape under homogeneously turbulent flow. They also generalized the expression for the turbulent diffusivity coefficient:

$$\varepsilon_p = K_e y^n.$$

(2)

In this expression, the parameter K_e is used to characterize the magnitude of the turbulent flow and the parameter n captures the effect of transverse distance from a surface (y) on the scale of turbulent eddies.

In the particle deposition literature, various values of the exponent n between 2 and ~ 3 have been reported. Many investigators have obtained noninteger values by fitting the Crump and Seinfeld model to their experimental data. In reviewing prior work, Chen *et al.* (1992), and Cheng (1997) note the discrepancies among various studies in the values of n and observe that the reasons are not understood.

Note that in equation (2) the turbulence parameter K_e has dimensions of $L^{2-n}T^{-1}$. When $n = 2$, as in equation (1), the dimensions of K_e are simply inverse time, but for other values of the exponent n , the dimensions of K_e must include a length component. Noninteger values of n , in particular, have caused conceptual and practical problems if the turbulence intensity parameter is to be estimated directly from information on turbulent energy dissipation rates. To resolve that problem, Beneš and Holub (1996) proposed a modified formulation of the Crump and Seinfeld model:

$$\varepsilon_p = K_e \delta^2 \left(\frac{y}{\delta}\right)^n,$$

(3)

where δ is a boundary-layer thickness. This formulation yields predictions that agree fairly well with experimental results in one study (Cheng, 1997). However, apart from eliminating

the problem of dimensional inconsistency associated with noninteger values of n , the expression lacks a strong physical foundation. The model includes three parameters (K_e , δ , and n), compared with two in the Crump and Seinfeld formulation (K_e and n) and only one in the original formulation by Corner and Pendlebury (K_e). How one would evaluate δ and n for arbitrary indoor environments remains unresolved.

Clearly, particle deposition is strongly affected by near surface air motion. Consequently, it is possible to gain insight into the process of turbulent particle deposition by studying the many experimental and computational studies of turbulent fluid motion near surfaces.

A widely accepted description considers that there are three zones in a turbulent boundary layer (Schlichting, 1979). A logical approach to analyze particle deposition from turbulent flow is to examine the turbulent structure zone by zone and formulate transport equations for each zone. The modeling advances presented in this paper follow that approach and yield a particle deposition model for indoor environments that is somewhat more complex than previous work, but still practical to use. As will be demonstrated, this approach helps to explain why a value of n of 2.6–2.8 in equation (2) is empirically observed to yield a good fit to experimental results. (It will be seen, however, that a value of $n = 2.95$ is more consistent with existing information on turbulence diffusivity.) The number of independent parameters in the new model is reduced to one, the friction velocity. Approaches for measuring the friction velocity in indoor air are briefly explored.

2. METHODS

2.1. Model formulation

We initially model particle deposition onto an isolated, smooth, isothermal and electrically neutral vertical wall. The effects of gravitational settling for deposition to horizontal surfaces will be included in Section 2.5.

The particle flux is assumed to be one-dimensional and steady. We derive the model for a fixed particle size (diameter). Sufficiently far from a surface, the air is assumed to be well mixed, so that a particle concentration gradient only exists very close to the surface. Because the Brownian diffusivity of particles is much smaller than the kinematic viscosity of air, the particle concentration boundary layer generally is contained within the viscous sublayer of the turbulent boundary layer. Two mechanisms are assumed to govern particle transport through the boundary layer to a vertical surface: Brownian diffusion and turbulent diffusion. (Other transport processes such as thermophoresis, electrostatic drift, and inertial drift are not included here.) Because low air velocities prevail indoors, the surface is assumed to be a perfect sink for deposition, i.e. no particle rebound or resuspension is considered. Within the boundary layer, there are no sources or sinks of particles, so the particle flux through the boundary layer, J , is constant throughout the particle concentration boundary layer. That flux can be described by a modified form of Fick's law:

$$J = -(\varepsilon_p + D) \frac{\partial C}{\partial y}, \quad (4)$$

where D is the Brownian diffusivity of the particle and C is the particle concentration in air.

Equation (4) has long been applied to evaluate particle deposition flux. Recently, however, researchers have modified equation (4) by adding a term to account for thermophoresis, a phenomenon that leads to the net migration of particles from regions of high to low eddy diffusivity (Reeks, 1983). The modified model results fit well to experimental data (Johansen, 1991; Guha, 1997; Young and Leeming, 1997). Nevertheless, this modification is only significant for particles with high inertia. The combination of air speeds and particle sizes of interest in indoor air is such that particle momentum in the boundary layer is small, and so equation (4) can be used without modification for the present purposes.

Letting C_∞ be the particle concentration outside the concentration boundary layer, the particle deposition velocity, v_d , is defined as

$$v_d = \frac{|J(y=0)|}{C_\infty}. \quad (5)$$

For convenience in model development, the particle concentration, distance from the surface, and deposition velocity are normalized by the freestream particle concentration, friction velocity and fluid kinematic viscosity, as follows:

$$C^+ = \frac{C}{C_\infty}, \quad (6)$$

$$y^+ = \frac{yu^*}{\nu}, \quad (7)$$

$$v_d^+ = \frac{v_d}{u^*}, \quad (8)$$

where ν is the kinematic viscosity of air and u^* is the friction velocity, defined by

$$u^* = \sqrt{\frac{\tau_w}{\rho_a}}. \quad (9)$$

Here, τ_w is the shear stress at the wall and ρ_a is the air density. The model is formulated so that the friction velocity is a key input parameter that captures the effects of indoor air flow intensity. The practical evaluation of u^* from indoor air velocity measurements is discussed in Section 2.6.

Using equations (5)–(8), along with the assumption that the flux is constant in the concentration boundary layer, equation (4) can be rewritten as

$$v_d^+ = \left(\frac{\varepsilon_p + D}{\nu} \right) \frac{\partial C^+}{\partial y^+}. \quad (10)$$

The deposition velocity is evaluated by rearranging equation (10) and integrating across the boundary layer, subject to the boundary conditions that $C^+ = 0$ at $y^+ = r^+$ and $C^+ = 1$ at $y^+ = 30$:

$$\frac{1}{v_d^+} = \int_0^1 \frac{dC^+}{v_d^+} = \int_{r^+}^{30} \left(\frac{\nu}{\varepsilon_p + D} \right) dy^+ = I. \quad (11)$$

Here, $r^+ = (d_p/2)(u^*/\nu)$, where d_p is the particle diameter. The boundary condition at $y^+ = r^+$ implies that the particle concentration is zero at the position where particles touch the surface. The second boundary condition assumes that the particle concentration is equal to the core value ($C = C_\infty$) at the outer edge of the fluid-mechanical buffer layer, $y^+ = 30$ (Bejan, 1995).

2.2. Particle eddy diffusivity in the boundary layer

Because of the physical constraints imposed by the surface, the particle eddy diffusivity within the boundary layer must vary with distance, from zero at the surface to some positive value at the edge of the particle concentration boundary layer. To proceed with the model, a functional form for $\varepsilon_p(y^+)$ must be sought.

We assume that the particle eddy diffusivity equals the fluid turbulent viscosity, ν_t . This assumption is justified by a general argument and by some recent numerical simulation results. Intuitively, the ratio of the particle eddy diffusivity to fluid turbulent viscosity should depend on the particle size and on some aspects of air flow. Hinze (1975) proved mathematically that in the long time limit, the particle eddy diffusivity is equal to the fluid turbulent viscosity in a homogeneously turbulent flow field. Fuchs (1964) made similar

arguments. Large particles with significant inertia respond less well to the turbulent fluctuations of their carrier fluid; however, their velocities are more persistent.

More recent numerical simulation studies also support this conclusion under limiting conditions. Inferring from the results of Uijttewaald and Oliemans (1996), the ratio ε_p/v_t can be taken as unity for dimensionless particle relaxation times τ^+ less than 0.1. The dimensionless particle relaxation time is defined by

$$\tau^+ = \tau \frac{u^{*2}}{v} = \left(\frac{C_c \rho_p d_p^2}{18 \rho_a v} \right) \left(\frac{u^{*2}}{v} \right), \quad (12)$$

where C_c is the slip correction factor and ρ_p is the particle density. An approximate upper bound for conditions of interest in indoor air is $\tau^+ \sim 0.05$, which occurs for $d_p = 10 \mu\text{m}$, $\rho_p = 2.5 \text{ g cm}^{-3}$, and $u^* = 3 \text{ cm s}^{-1}$.

Strictly, the conclusion that $\varepsilon_p/v_t = 1$ should only be valid for homogeneous isotropic turbulence. For particle deposition, the rate-limiting transport processes occur inside the viscous sublayer, which is known to be anisotropic (Young and Leeming, 1997). In this case, particles might retain their earlier motion because of inertia into a region that has different turbulence properties. Nevertheless, because of the small dimensionless particle relaxation times encountered in real indoor environments, the ratio can still be assumed to equal unity. Simulation results by Kallio and Reeks (1989) showed that even for τ^+ up to the order of 10, the velocity of the fluid parcel and of particles inside the sublayer was almost the same.

2.3. Fluid turbulent viscosity within the boundary layer

Now that we have concluded that $\varepsilon_p = v_t$, the next step is to obtain an expression for the dependence of fluid turbulent viscosity on distance from the surface. In principle, the data for such an expression can either be obtained from experimental investigations or from numerical simulations.

Fluid turbulent viscosity can be measured directly by an eddy correlation technique, which requires measurement of the components of fluctuating fluid velocity near a surface. The most advanced experimental measurements by laser Doppler velocimetry (LDV) can measure the velocity components to within $y^+ \sim 2$ from a surface (Fontaine and Deutsch, 1995). Closer to the surface, the LDV signal is degraded.

With the advance of computer technology, direct numerical simulation (DNS) of turbulence has become possible. In DNS simulation, full resolution of all eddy scales is performed. Since no *ad hoc* models are needed, DNS results, unlike results from statistical simulations or large eddy simulation, essentially are free of error due to empirical assumptions about the intrinsic turbulent physics (Härtel, 1996). Kim *et al.* (1987) reported fluctuating velocity components based on DNS to within a distance $y^+ = 0.05$ of a surface. These simulation results are widely regarded as a benchmark (Balint *et al.*, 1991). However the flow examined by Kim *et al.* was for a channel. We must justify the application of their results to the turbulent boundary layer adjacent to surfaces in an enclosure. Spalart (1988) performed DNS for a turbulent boundary layer with different Reynolds numbers. The lowest Reynolds number in his simulations was 300, based on the friction velocity and momentum boundary layer thickness. In our modeling efforts, an (upper) bound estimate of Re using the same basis is 120. Close examination of the results of Kim *et al.* (1987) and Spalart (1988) indicates a strong similarity. Fontaine and Deutsch (1995) commented that for $y^+ < 30$, the turbulent statistics of different flow geometries are similar for equal Reynolds numbers. An additional reason that favors the use of the results of Kim *et al.* is the similarity of the ratio of characteristic mean velocity to the friction velocity (15.6 for Kim *et al.* and 14.5 for a typical present case).

With some caution, then, turbulent fluid viscosity can be evaluated by making use of the DNS results of Kim *et al.* (1987, Table 4.2). Figure 1 shows the ratio of turbulent to molecular viscosity (v_t/v) for $y^+ \leq 30$. To facilitate carrying out the integration in equation (11), we directly fit the DNS results of Kim *et al.* by power-law expressions. The resulting

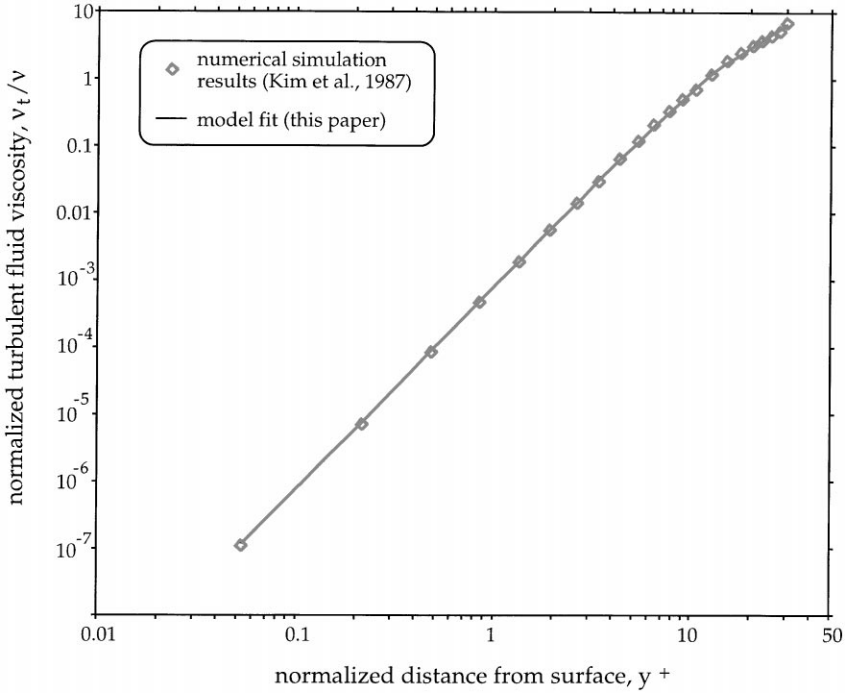


Fig. 1. Dimensionless eddy viscosity versus dimensionless distance from a surface. The model fit is presented in equation (13).

equations, illustrated in Fig. 1, are

$$\frac{v_t}{v} = 7.669 \times 10^{-4} (y^+)^3, \quad 0 \leq y^+ \leq 4.3, \quad (13a)$$

$$\frac{v_t}{v} = 1.00 \times 10^{-3} (y^+)^{2.8214}, \quad 4.3 \leq y^+ \leq 12.5, \quad (13b)$$

$$\frac{v_t}{v} = 1.07 \times 10^{-2} (y^+)^{1.8895}, \quad 12.5 \leq y^+ \leq 30. \quad (13c)$$

As shown in Fig. 1, the DNS results are successfully described by equations (13) without significant discontinuities. Because of the low Brownian diffusivity of particles, the particle concentration boundary layer is generally contained within the viscous sublayer (see Fig. 2). Therefore, the functional dependence of turbulent diffusivity on y^+ is most important in the region $0 \leq y^+ \leq 4.3$. The cube power relationship reported here (equation (13a)) has been observed in simulations and experiments (Chapman and Kuhn, 1986; Karlsson and Johansson, 1988; Meneveau *et al.*, 1996).

2.4. Particle deposition velocity for vertical surfaces

The deposition velocity can now be obtained by substituting equations (13), with $\varepsilon_p = v_t$, into equation (11) and then integrating over the three y^+ intervals. To obtain an analytical expression, the integral for the outer two layers ($y^+ \geq 4.3$) is simplified by assuming that Brownian diffusivity can be ignored relative to the much larger turbulent diffusivity ($D \ll \varepsilon_p$). This approximation is valid to within 1% for particle diameters larger than $0.01 \mu\text{m}$. For smaller particles, equation (11) must be integrated numerically, including Brownian diffusivity, through each layer. The resulting equations and numerical results for evaluating deposition velocity as a function of particle size and fluid friction velocity are presented in Tables 2 and 3.

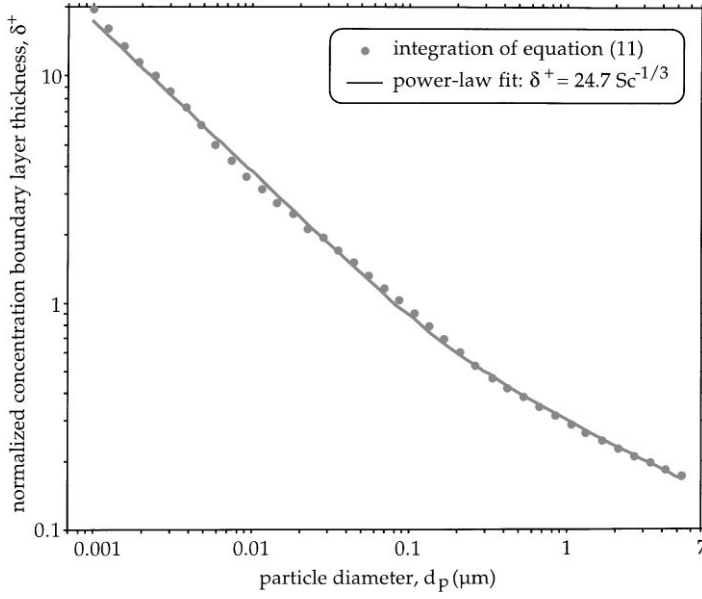


Fig. 2. Dimensionless concentration boundary layer thickness as a function of particle diameter. The boundary layer thickness is defined to be the distance from the wall at which $C = 0.9 C_\infty$; $\delta^+ = \delta u^* v^{-1}$. The Schmidt number is $Sc = \nu/D$.

Figure 2 demonstrates the importance of estimating the fluid turbulent viscosity very near the wall by showing the predicted concentration boundary layer thickness (defined as $\delta^+ = y^+$ such that $C^+ = 0.9$) as a function of particle diameter. The results reveal that even for particles as small as $0.01 \mu\text{m}$, the boundary layer thickness is contained within the viscous sublayer ($y^+ \leq 4.3$). Given a typical indoor friction velocity of $u^* = 1 \text{ cm s}^{-1}$, the boundary layer thickness for a $0.3 \mu\text{m}$ particle would be 0.07 cm . As expected, the thickness of the particle concentration boundary layer diminishes significantly with increasing particle diameter. The boundary layer thickness can be described by a simple power-law dependence on the Schmidt number ($Sc = \nu/D$):

$$\delta^+ = 24.7 Sc^{-1/3}. \quad (14)$$

2.5. Particle deposition velocity for horizontal surfaces

For particles larger than $\sim 0.1 \mu\text{m}$, any model for deposition to a horizontal (or nonvertical) surface must account for the influence of gravity. Given a settling velocity v_s , the equation for particle flux within the boundary layer adjacent to a horizontal surface can be written as

$$J = -(D + \varepsilon_p) \frac{dC}{dy} \pm v_s C, \quad (15)$$

where for the settling term, the positive sign applies for a downward-facing surface and the negative sign applies for an upward-facing surface. The deposition velocity is obtained by following same procedure outlined in the previous sections, with these results:

$$v_d^+ = \frac{v_s^+}{1 - \exp(-v_s^+ I)}, \quad \text{upward surface}, \quad (16a)$$

$$v_d^+ = \frac{v_s^+}{\exp(-v_s^+ I) - 1}, \quad \text{downward surface}. \quad (16b)$$

Table 2. Summary of equations for predicting particle deposition according to present model*

Parameter	Equation
Integral†	$I = [3.64 \text{ Sc}^{2/3} (a - b) + 39]$ $a = \frac{1}{2} \ln \left[\frac{(10.92 \text{ Sc}^{-1/3} + 4.3)^3}{\text{Sc}^{-1} + 0.0609} \right] + \sqrt{3} \tan^{-1} \left[\frac{8.6 - 10.92 \text{ Sc}^{-1/3}}{\sqrt{3} \text{ } 10.92 \text{ Sc}^{-1/3}} \right]$ $b = \frac{1}{2} \ln \left[\frac{(10.92 \text{ Sc}^{-1/3} + r^+)^3}{\text{Sc}^{-1} + 7.669 \times 10^{-4} (r^+)^3} \right] + \sqrt{3} \tan^{-1} \left[\frac{2r^+ - 10.92 \text{ Sc}^{-1/3}}{\sqrt{3} \text{ } 10.92 \text{ Sc}^{-1/3}} \right]$
Deposition velocity, vertical surface	$v_{dv} = \frac{u^*}{I}$
Deposition velocity, upward horizontal surface	$v_{du} = \frac{v_s}{1 - \exp\left(-\frac{v_s I}{u^*}\right)}$
Deposition velocity, downward horizontal surface	$v_{dd} = \frac{v_s}{\exp\left(\frac{v_s I}{u^*}\right) - 1}$
First-order loss coefficient for deposition, rectangular cavity	$\beta = \frac{v_{dv} A_v + v_{du} A_u + v_{dd} A_d}{V}$
First-order loss coefficient for deposition, spherical cavity‡	$\beta = \frac{3}{2R} \left(\frac{u^*}{I} \right) \left[2D_1(x) + \frac{1}{2}x \right] \quad \text{where } x = \frac{v_s I}{u^*}$

* Nomenclature: $\text{Sc} = \nu D^{-1}$ where ν is the kinematic viscosity of air and D is the Brownian diffusivity of the particle; $r^+ = d_p u^*(2\nu)^{-1}$, where d_p is particle diameter; u^* is friction velocity; v_s = gravitational setting velocity of particle; A_v = area of vertical surfaces; A_u = area of upward-facing surfaces; A_d = area of downward-facing surfaces; V = room volume; R = radius of spherical enclosure, and $D_1(x)$ is the Debye function:

$$D_1(x) = \frac{1}{x} \int_0^x \frac{t \, dt}{e^t - 1}.$$

† The integral is evaluated analytically under the approximation that the Brownian diffusivity, D , is negligible compared with eddy diffusivity for $y^+ \geq 4.3$. The approximation is accurate to 1% or better for particle diameters larger than $0.01 \, \mu\text{m}$. For smaller particles, the integration of equation (11) must be carried out numerically. See Table 3 for results.

‡ In the limit of small particles (negligible influence of gravitational settling) the expression simplifies to $\beta = 3u^*(RI)^{-1}$. In the limit of large particles (negligible influence of Brownian diffusion) the expression simplifies to $\beta = 3v_s(4R)^{-1}$.

Table 3. Results from numerical integration of equation (11) for very fine particles

Particle diameter, d_p (μm)	Integral, I (—)
0.001	29.1
0.0015	49.1
0.002	71.0
0.003	120.3
0.004	174.9
0.005	234.2
0.006	297.4
0.007	364.0
0.008	432.7
0.009	504.5
0.01	579.3

The dimensionless settling velocity is given by

$$v_s^+ = \frac{v_s}{u^*}. \quad (17)$$

The integral I is defined by the right-hand portion of equation (11); its evaluation is presented in Tables 2 and 3.

2.6. Estimating the friction velocity

To apply our model for predicting the dimensional deposition velocity, v_d , the friction velocity must be evaluated. This is the only parameter used in the model to characterize the nature and intensity of near-surface turbulent flow.

The shear stress can be expressed as

$$\tau_w = \rho_a v \left. \frac{dU}{dy} \right|_{y=0}, \quad (18)$$

where U is the mean air speed parallel to the surface. Substituting into equation (9) yields

$$u^* = \left(v \left. \frac{dU}{dy} \right|_{y=0} \right)^{1/2}. \quad (19)$$

To evaluate the friction velocity, therefore, we need to know the velocity gradient at the wall. One means for estimating this is based on balancing the momentum and skin friction forces on a smooth plate immersed in a turbulent flow field. Given a freestream velocity U_∞ and plate length L , the average velocity gradient is (see, e.g. Schlichting, 1979)

$$\left. \frac{dU}{dy} \right|_{y=0} = \left(\frac{0.074}{\rho_a v} \right) \left(\frac{\rho_a U_\infty^2}{2} \right) \left(\frac{U_\infty L}{v} \right)^{-1/5}. \quad (20)$$

Equations (19) and (20) can be applied to estimate friction velocity based on a representative freestream air velocity measurement, U_∞ , and a characteristic length of room surfaces, L .

Another way to determine friction velocity is based on measurement of the velocity profile in the logarithmic flow region near a surface. According to the “law of the wall”, in the portion of the turbulent boundary layer where $v_t \gg v$, the time-averaged velocity varies with distance from the surface according to (Bejan, 1995)

$$u^+ = A \ln(y^+) + B, \quad (21)$$

where A and B are constants whose respective values are approximately 2.5 and 5–5.5. The dimensionless velocity u^+ is the time-averaged air speed parallel to the surface, normalized by the friction velocity, u^* . Substituting $A = 2.5$, equation (21) can be rewritten as

$$\frac{U}{U_\infty} = \frac{2.5u^*}{U_\infty} \ln \left(\frac{yU_\infty}{v} \right) + D, \quad (22)$$

where U is the time-averaged velocity parallel to the surface, and D is a constant. By plotting measurements of U/U_∞ versus the logarithm of $(yU_\infty v^{-1})$, the friction velocity can be obtained from the slope of the line. This approach is known as the Clauser-plot method (Brunn, 1995).

Because of the low air speeds indoors, it is not trivial to measure the velocity profile near indoor surfaces. Traditional hot-wire probes cannot measure velocity accurately at speeds less than 5–10 cm s⁻¹. By reducing the applied voltage across the hot wire, Zhang *et al.* (1995) improved hot-wire sensitivity for measuring near-surface flow profiles in rooms. Figure 3 depicts one set of results from their study presented in the Clauser-plot format. For these data, application of equation (22) shows that the friction velocity is 2.6 cm s⁻¹.

Laser-doppler velocimetry, which can measure very low flow velocities with high precision and very good spatial and temporal resolution appears to be an attractive method of measuring friction velocities indoors. Further experimental research effort is needed to demonstrate this.

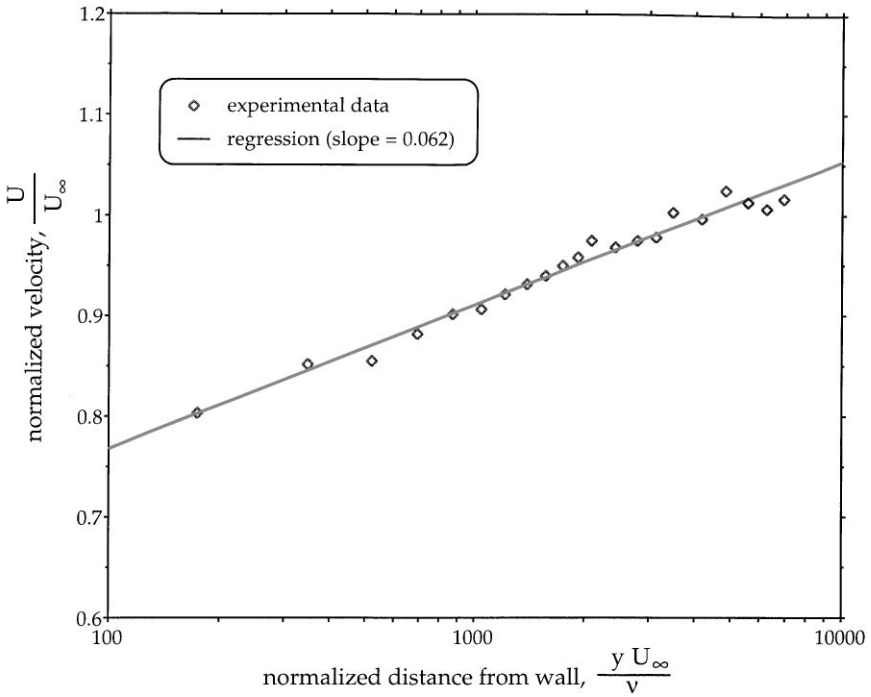


Fig. 3. Estimation of friction velocity in an office from near-wall velocity measurements. Experimental data are from Zhang *et al.* (1995). The friction velocity estimated from these data is $u^* = 2.6 \text{ cm s}^{-1}$.

3. RESULTS AND DISCUSSION

To illustrate the results of this model, we focus on a room in a typical office building with air flow supplied by mechanical ventilation. Measurements have shown that air motion in ordinary mechanically ventilated spaces is turbulent (Etheridge and Sandberg, 1996). Such a room might have a typical mean air speed of $U_\infty = 15 \text{ cm s}^{-1}$ (Furtaw *et al.*, 1996) and a characteristic dimension of $L = 3 \text{ m}$ (assumes a volume of $V \sim 30 \text{ m}^3$ and $L \sim V^{1/3}$). According to equations (19)–(20), the corresponding surface shear rate (dU/dy at $y = 0$) would be 7 s^{-1} and the friction velocity would be $u^* = 1.0 \text{ cm s}^{-1}$. Figure 4 shows the deposition velocity as a function of particle diameter predicted for a representative range of friction velocities ($0.3\text{--}3 \text{ cm s}^{-1}$) for (a) a vertical wall, (b) the floor and (c) the ceiling.

It is instructive to compare the results of the present model with those of Corner and Pendlebury (1951) and Crump and Seinfeld (1981). The key variable characterizing flow in the Corner and Pendlebury model is the turbulence intensity parameter, K_e , which is related to the surface shear rate by the expression

$$K_e = \kappa^2 \frac{dU}{dy}, \quad (23)$$

where κ is the von Kármán constant, assumed to be 0.4 for the present comparison. The typical shear rate of 7 s^{-1} cited above corresponds to a turbulence intensity parameter of $K_e = 1.1 \text{ s}^{-1}$, within the range $0.1\text{--}10 \text{ s}^{-1}$ previously cited as encompassing typical indoor circumstances (Nazaroff and Cass, 1989; see also Nazaroff *et al.*, 1990, Table 3).

To predict the overall particle loss rate by deposition to room surfaces, we follow the same approach described by Corner and Pendlebury (1951): deposition to all surfaces is assumed to proceed independently, in parallel. Thus, the overall first-order loss rate coefficient caused by deposition to room surfaces is

$$\beta = \frac{v_{dv}A_v + v_{du}A_u + v_{dd}A_d}{V}, \quad (24)$$

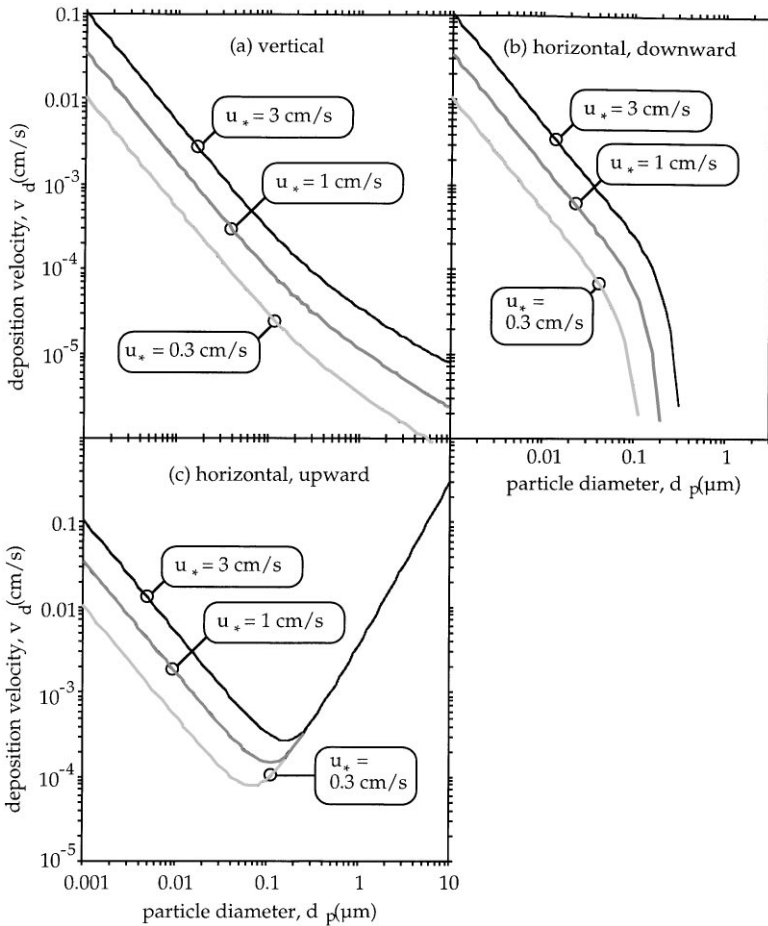


Fig. 4. Deposition velocity as a function of particle diameter and friction velocity: (a) vertical wall, (b) downward-facing horizontal surface and (c) upward-facing horizontal surface. Predictions assume air pressure is 1 atm, temperature is 293 K and particle density is 1.0 g cm^{-3} .

where v_{dv} , v_{du} , and v_{dd} represent the deposition velocities to the vertical, upward-facing, and downward-facing surfaces, respectively; A_v , A_u , and A_d are the total deposition areas of the vertical, upward, and downward surfaces; and V is the room volume. Figure 5 shows the deposition rate loss rate coefficient, β , predicted by the present model.

The predictions presented in Fig. 5 agree well with the Crump and Seinfeld (1981) model with appropriate selection of K_e and n in equation (2). For example, with the least-squares fitted parameters, $n = 2.95$ and $K_e = 0.784 \text{ cm}^{-0.95} \text{ s}^{-1}$, the maximum deviation between β predicted by Crump and Seinfeld and by the current model with $u^* = 3 \text{ cm s}^{-1}$ is 2.6% over the range $0.001 \mu\text{m} \leq d_p \leq 10 \mu\text{m}$. Similarly, good agreement can be achieved for $u^* = 1 \text{ cm s}^{-1}$ ($n = 2.94$ and $K_e = 0.0305 \text{ cm}^{-0.94} \text{ s}^{-1}$; maximum error = 2.2%) and $u^* = 0.3 \text{ cm s}^{-1}$ ($n = 2.94$ and $K_e = 0.00089 \text{ cm}^{-0.94} \text{ s}^{-1}$; maximum error = 2.3%).

Comparisons of the Crump and Seinfeld model with experimental data in chamber studies show that the best-fit value of the exponent n is 2.6–2.8 (Cheng, 1997). Although our model agrees best with Crump and Seinfeld with $n = 2.94$ –2.95, the dependence of the fit on n is not extremely strong. For example, with $u^* = 1 \text{ cm s}^{-1}$, the Crump and Seinfeld model can be adjusted to fit the current model with a maximum deviation of 19% for $n = 2.6$ ($K_e = 0.0207 \text{ cm}^{-0.6} \text{ s}^{-1}$) or with a maximum deviation of 7% for $n = 2.8$ ($K_e = 0.0255 \text{ cm}^{-0.8} \text{ s}^{-1}$). It would be unusual for experimental data on particle loss rate coefficients to have less uncertainty than this. Also, the assumptions of smooth surfaces may not be fully met experimentally. Any irregularities in surface geometry, such as surface roughness, would tend to increase deposition of accumulation mode particles by a larger factor

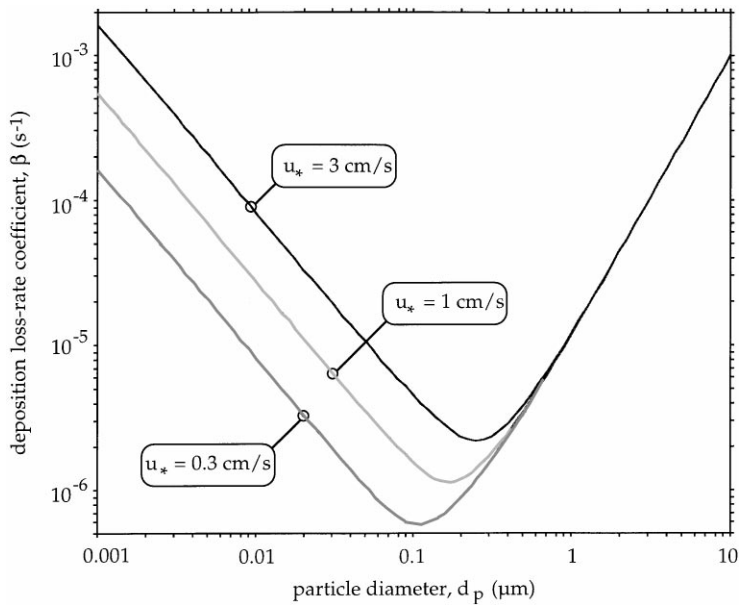


Fig. 5. Particle deposition loss-rate coefficient, β , for typical room dimensions (3 m high $\times$ 4 m $\times$ 5 m) according to the current model. Friction velocities of 0.3–3 cm s⁻¹ approximately span the range expected for mechanically ventilated indoor spaces. Predictions assume air pressure is 1 atm, temperature is 293 K and particle density is 1.0 g cm⁻³.

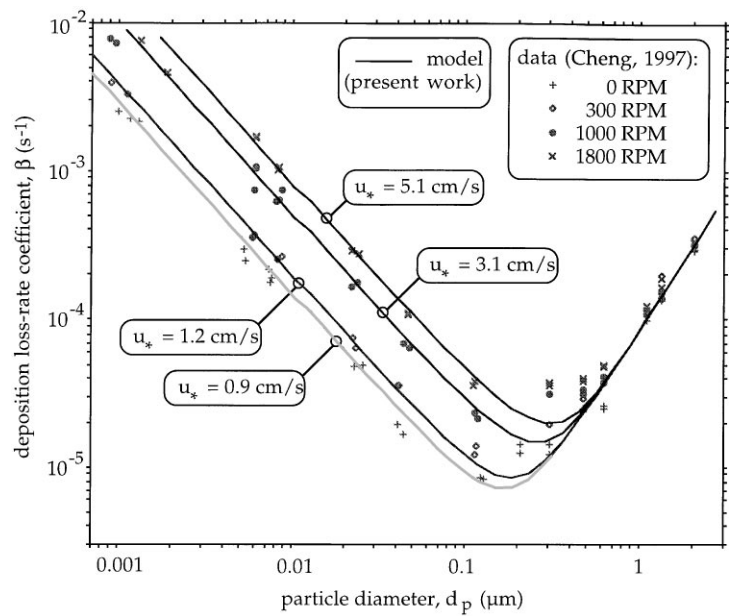


Fig. 6. Comparison of experimental data (Cheng, 1997) with predictions from the current model for the particle loss-rate coefficient, β , as a function of particle diameter, d_p , in a 161 L aluminum sphere. The friction velocity is treated as a fitted parameter. The equations for predicting loss rate in a spherical cavity are presented in Table 2. The predictions assume air pressure is 0.8 atm, temperature is 295 K and particle density is 1.0 g cm⁻³.

than nucleation mode particles (Shimada *et al.*, 1988), leading to smaller best-fit values of n . Note that the best-fit result of $n = 2.94\text{--}2.95$ is very close to $n = 3$, which is commonly used to describe turbulent heat transfer (Pandian and Friedlander, 1988). Since particle inertia is not an important deposition mechanism indoors for small particles, the analogy between heat and mass transfer should hold.

Figure 6 compares predictions of the present model with experimental data from Cheng (1997). In this case, our model has been reformulated for a spherical cavity (see Table 2). Lacking independent data, the friction velocity is treated as a fitting parameter. The agreement between experiment and model is seen to be very good over the full range of particle sizes. The largest discrepancies are seen in the size range $0.1 \mu\text{m} \leq d_p \leq 1 \mu\text{m}$. These discrepancies may be a result of deviations of the chamber surface from the model assumption of a perfectly smooth sphere.

The model developments presented here have focused on improving the description of turbulent transport through the particle concentration boundary layer near surfaces. For some situations, it may be important to include other transport mechanisms, such as thermophoresis (Nazaroff and Cass, 1989), electrostatic drift (McMurry and Rader, 1985), and turbophoresis (Guha, 1997; Young and Leeming, 1997). Also, this work has only considered deposition onto smooth surfaces. We intend to address the role of surface roughness on indoor particle deposition in a later paper.

A key parameter used in this study, the friction velocity, merits further discussion. This normalization parameter is commonly used to describe turbulent flows in channels, pipes, and boundary layers. Because it is defined in terms of the shear stress on a surface (equation (9)), it is only meaningful when the flow has a prevailing direction. Commonly, this condition will be satisfied along major portions of indoor surfaces because of the strong flow induced by air discharge from supply registers. However, for some situations, a prevailing flow direction may be weak or nonexistent. For such cases, another normalization parameter may be necessary to describe the air flow conditions. We speculate that an appropriate parameter in such cases is the root-mean-square velocity fluctuation. Further study is needed to test this idea.

4. CONCLUSION

Deposition onto surfaces is an important fate for particles in indoor air. Models of particle deposition from turbulent flow onto enclosure surfaces have been developed and applied since the early 1950s. In this paper, we have contributed to this evolution by incorporating recent information on the structure of turbulent diffusivity near surfaces. This is an important step towards gaining a complete mechanistic understanding of particle deposition indoors. The result is a mathematical model that remains relatively easy to use. The model yields predictions that are consistent with the widely used model of Crump and Seinfeld. The best fit occurs with $n = 2.95$, but good agreement is obtained with $n = 2.8$. Small surface irregularities could cause this empirically determined exponent to shift by enhancing the rate of deposition of accumulation mode particles by a larger factor than for nucleation mode particles.

For practical application of this modeling approach in indoor air, additional work is needed to account for the effects of surface irregularities such as roughness. Continued development of methods for evaluating the friction velocity based on readily measured variables is also needed. To further validate the model, it would also be of great benefit to have the results of carefully conducted experiments in full-scale rooms in which particle deposition as a function of size and near-surface air flow conditions was well characterized.

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